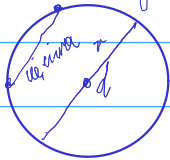


T: pzo

T: Okrąg

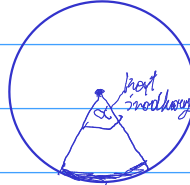
Okrąg



$$d = 2r$$

średnica
dl. okręgu $L = 2\pi r$

$$\frac{dl. \text{ okręgu}}{\text{średnica okręgu}} = \pi \approx 3,14$$



$$L = \frac{\alpha}{360^\circ} \cdot 2\pi r$$

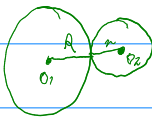
$$\frac{\alpha}{360^\circ} = \frac{\text{całki kąt środkowy}}{360^\circ}$$

dlugość łuku okręgu

Rodzaje okręgów ze względu na ich położenie

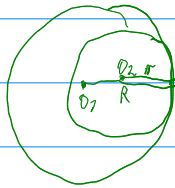
1 punkt wspólny

styczne zewnętrznie



$$|O_1 O_2| = R + r$$

styczne wewnętrznie



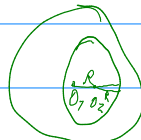
$$|O_1 O_2| = |R - r|$$

0 punkt wspólny



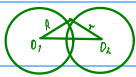
$$|O_1 O_2| > R + r$$

rozłączne wewnętrznie



$$|O_1 O_2| < |R - r|$$

2 punkty wspólne



$$R + r > |O_1 O_2| > |R - r|$$

Cw 2 str 229



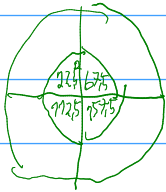
$$L = \frac{\alpha}{360^\circ} \cdot 2\pi \cdot 12$$

$$\alpha = 360^\circ \cdot \frac{3\pi}{2\pi \cdot 12} = 360^\circ \cdot \frac{1}{8} = 45^\circ$$

Ćw 2 An 2.25

$$d = 24 \text{ cm}$$

$$r = 12 \text{ cm}$$



$$L = 2\pi r = 2 \cdot \pi \cdot 12 = 24\pi$$

$$d_1 = \frac{22.5}{360} \cdot d = 10.5\pi$$

$$d_2 = \frac{112.5}{360} \cdot L = 4.5\pi$$

$$d_3 = \frac{22.5}{360} \cdot L = 4.5\pi$$

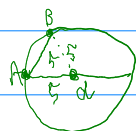
$$d_3 = \frac{112.5}{360} \cdot L = 7.5\pi$$

Ćw 3

$$|O_1 O_2| = 48$$

$$d = 16$$

Ćw 1 An 2.26

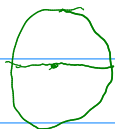


$$d = 10$$

$$\frac{\alpha}{360} \cdot 2\pi r$$

$$\frac{60}{360} \cdot 2\pi \cdot 5 = \frac{1}{6} \cdot 10\pi = \frac{10}{6}\pi$$

Ćw 3 An 2.26



$$L = 2\pi \cdot 18 = 36\pi$$

$$2\pi r = 36\pi$$

$$r = 18$$

$$d = 36$$

Ćw 5 An 2.26



$$P(x, 0) \quad Q(-7, 9)$$

$$|PQ| = 8$$

$$x = 8 + (-7) = 1$$

$$x = -7 - 9 = -16$$

$$x = 7 - 9 = -2$$



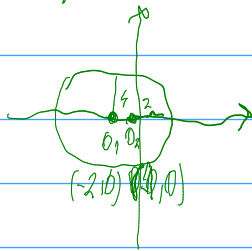
$$|PQ| = 5 - 3 = 2$$

$$x = -7 - 2 = -9$$

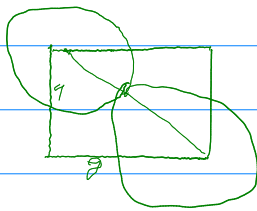
$$x = -7 + 2 = -5$$

$$x = -3 \vee 7$$

Zad 6/226



Zad 8/226



$$4^2 + 8^2 = x^2$$

$$16 + 64 = x^2$$

$$x = 4\sqrt{5}$$

$$1x + 3x = 4\sqrt{5}$$

$$4x = 4\sqrt{5}$$

$$x = \sqrt{5}$$

$$L_1 = 2\pi r = 2\pi\sqrt{5}$$

$$L_2 = 6\pi\sqrt{5}$$

T: koło



$$P_K = \pi r^2$$

Różnica powierzchni koła



$$P_{\text{koła}} = \frac{\alpha}{360} \cdot \pi r^2$$

Pierścień kołowy



$$P_K = P_{K1} - P_{K2}$$

$$\frac{\alpha}{360} = \frac{P_{\text{koła}}}{\pi r^2}$$

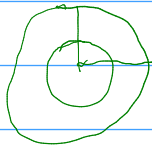
Dobitek kołowy



$$P = P_{K1} - P_{\Delta}$$

Üw 2/227

a)



$$R = 25$$

$$r = 24$$

$$P_1 = \pi 25^2 = \pi 625$$

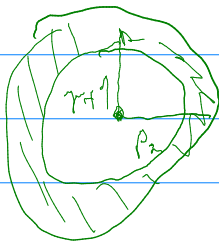
$$P_2 = \pi 24^2 = \pi 576$$

$$P_{\text{Wk}} = 49\pi$$

$$\pi r^2 = 49\pi$$

$$r = 7$$

b)



$$P_1 = 17\pi \text{ cm}^2$$

$$P_1 = \pi (r+1)^2$$

$$P_2 = \pi r^2$$

$$\pi r^2 + \pi (r+1)^2 = 17\pi$$

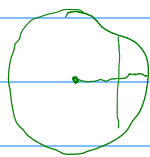
$$(r+1)^2 + r^2 = 17$$

$$r^2 + 2r + 1 + r^2 = 17$$

$$2r + 1 = 17$$

$$r = 8$$

Üw 4/228



$$r = 6$$

$$P_{\text{Wk}} = 2\pi$$

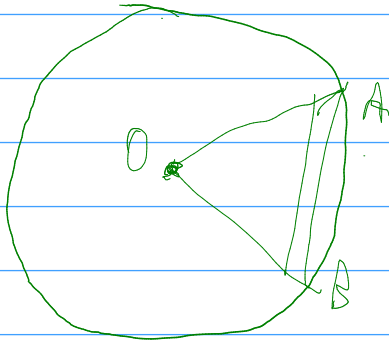
$$2\pi = \frac{\alpha}{360^\circ} \cdot 36\pi$$

$$\alpha = \frac{2\pi}{36\pi} \cdot 360^\circ = 20^\circ$$

$$\alpha = 20^\circ$$

Ex 5/228

a



$$r=12$$

$$\alpha = 90^\circ$$

$$\frac{900}{3600} \pi 12^2$$

$$P = \frac{1}{2} \cdot a \cdot b \cdot \sin \alpha$$

$$36\pi = P_{\text{out}}$$

$$P_{\text{in}} = \frac{1}{2} \cdot 12 \cdot 12 \cdot \sin 90^\circ = 72$$

$$b) P = \frac{1}{2} \cdot a \cdot b \cdot \sin \alpha$$

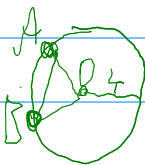
$$P = \frac{144}{2} \cdot 0.15$$

$$P = 36$$

$$P_{\text{work}} = 36 \cdot 12 \pi = 36$$

Ex 5/228

2nd 3/228



$$P = 4\sqrt{2}$$

$$P = \pi r^2$$

$$P = 16\pi$$

$$P = 16\pi - 4\sqrt{2}$$

$$P_{\text{in}} = P$$

$$P_{\text{work}} = \frac{90}{360} \cdot 16\pi = \frac{4}{9} \cdot 16\pi = 2\pi$$

$$P_{\text{work}} - P_{\text{in}} = 2\pi - 4\sqrt{2}$$

$$4\sqrt{2} = \frac{1}{2} \cdot 4 \cdot 4 \cdot \sin \alpha$$

$$4\sqrt{2} = 8 \sin \alpha$$

$$\sqrt{2} = 2 \sin \alpha$$

$$\frac{\sqrt{2}}{2} = \sin \alpha$$

$$\sin \alpha = \frac{\sqrt{2}}{2}$$

2.2.2.5/2.2.8

a) $360 \approx 8 \times 280$

$x = 10$

$P_{WK} \approx 2\pi$

b) $\alpha_s = 30^\circ$

$P_{\alpha 3} = \frac{30}{360} \cdot 9\pi$ $\alpha_6 = 60^\circ$ $P_{\alpha 6} = \frac{60}{360} \cdot 9\pi$

$P_{\alpha 3} = \frac{1}{12} \cdot 9\pi$ $P_{\alpha 6} = \frac{1}{6} \cdot 9\pi$

$P_{\alpha 3} = \frac{3}{4}\pi$

$\frac{1}{2} \cdot 3 \cdot 3 \cdot \sin 30^\circ = \frac{9}{2} \cdot \frac{1}{2} = \frac{9}{4}$

$P_1 = \frac{3}{4} = \frac{9}{4}$

2.2.2.6/2.2.9



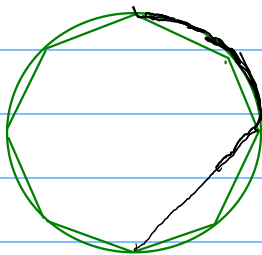
$\frac{1}{2} \cdot 2 \cdot 2 \cdot \sin 120^\circ = 2 \cdot \sin 60^\circ = 2 \cdot \frac{\sqrt{3}}{2} = \sqrt{3}$



$L = \frac{\alpha}{360} \cdot 2\pi r = \frac{120}{360} \cdot 2\pi \cdot 2 = \frac{1}{3} \cdot 4\pi = \frac{4\pi}{3}$

$P_{WK} = \frac{\alpha}{360} \cdot \pi r^2 = \frac{120}{360} \cdot \pi \cdot 4 = \frac{4\pi}{3}$
 $P_{\text{odkret}} = \left(\frac{4\pi}{3} - \sqrt{3}\right) \cdot 3 = 4\pi - 3\sqrt{3}$

c)



$\alpha = 65^\circ$

$L = \frac{\alpha}{360} \cdot 2\pi r = \frac{65}{360} \cdot 2\pi \cdot 2 = \frac{13}{36} \pi$ $P_{WK} \geq \frac{65}{360} \cdot 4\pi = \frac{13}{18} \pi$

$b^2 + a^2 = r^2$

$b^2 = 8$

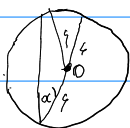
$b = 2\sqrt{2}$

$P = \frac{1}{2} \cdot 2 \cdot 2 \cdot \frac{\sqrt{2}}{2} = \frac{4\sqrt{2}}{2} = 2\sqrt{2}$

$P_{WK} = \frac{13}{18} \pi - 2\sqrt{2}$

$P \approx 8 \left(\frac{1}{2} \pi - \sqrt{2}\right)$

2.2.7/2.2.9



t: wzajemne połozenie okręgu i prostej

0 pkt wspólnych



$$|OP| > r$$

1 pkt wspólny

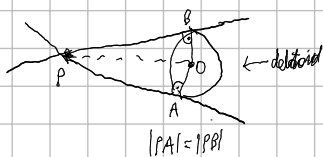


$$|OP| = r$$

2 pkt wspólne



Wzór odczytany



$$|PA| = |PB|$$

Ćw 2/230

a)



$$\begin{aligned} a^2 + b^2 &= c^2 \\ 6^2 + 4^2 &= c^2 \\ c &= 2\sqrt{5} \end{aligned}$$

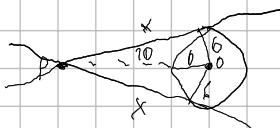
b)



$$\begin{aligned} 16^2 + 12^2 &= 20^2 \\ 400 - 256 &= 144 \\ 144 &= 12^2 \\ 12 &= 12 \end{aligned}$$

$$r = 12$$

Ćw 3/230



$$\begin{aligned} 10^2 &= x^2 + 6^2 \\ 100 &= x^2 + 36 \\ x &= 8 \end{aligned}$$

$$\text{Odcinek: } 12 + 16 = 28$$

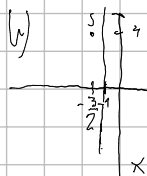
Ćw 4

b)



3 aty same

Zad 5/231



1 pkt $a = \frac{1}{2}$
0 pkt $r \in (0, \frac{1}{2})$
2 pkt $r > \frac{1}{2}$

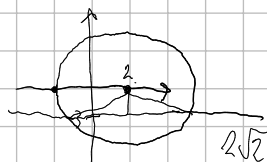
c)



0 pkt poprawa $(\sqrt{2}; 0)$
1 pkt poprawa $x = \sqrt{2} + 1$
2 pkt poprawa $r > \sqrt{2} + 1$

Ćw 6/233

a)



$$\begin{aligned} 4^2 + 3^2 &= x^2 \\ 16 + 9 &= x^2 \\ x &= \sqrt{25} \\ x &= 5 \end{aligned}$$

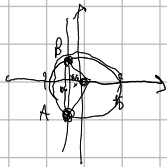
b)



$$\begin{aligned} 4^2 + 3^2 &= c^2 \\ 16 + 9 &= c^2 \\ c &= 5 \end{aligned}$$

Hand 1 str 233

c)

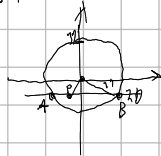


$$\begin{aligned} 5^2 + x^2 &= 10^2 \\ x^2 &= 75 \\ x &= 5\sqrt{3} \\ x &\approx 5\sqrt{3} \quad \vee x = 5\sqrt{3} \end{aligned}$$

Hand 2

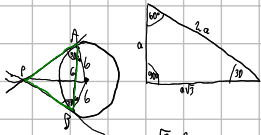


Hand 4



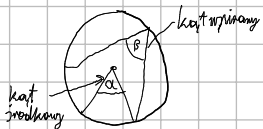
$$\begin{aligned} 11^2 &= 10^2 + x^2 \\ 121 - 100 &= x^2 \\ x^2 &= 21 \\ x &= \sqrt{21} \\ 5^2 &= (\sqrt{21})^2 + x^2 \\ 25 &= x^2 \\ x &= \end{aligned}$$

Zonal 6



$$\begin{aligned} a\sqrt{3} &= 6 \\ a &= \frac{6}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = 2\sqrt{3} \\ P_{\text{air}} &= \frac{1}{2} (2\sqrt{3} \cdot 6) = 6\sqrt{3} \\ P_{\text{air}} &= \frac{1}{2} \cdot 6 \cdot 6 \cdot \frac{\sqrt{3}}{2} = 9\sqrt{3} \\ P_{\text{total}} &= 12\sqrt{3} - 9\sqrt{3} = 3\sqrt{3} \end{aligned}$$

Ti kaiti waka



1)



2)



3)



4) tur, o cigenach



$$\begin{aligned} \frac{a}{b} &= \frac{c}{d} \\ x \cdot y &= z \cdot w \\ |PA| \cdot |PB| &= |PC| \cdot |PD| \end{aligned}$$

5)

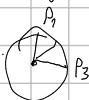


Hand 1 str 235

a) $\alpha = 60^\circ$

w) 240

c) 330

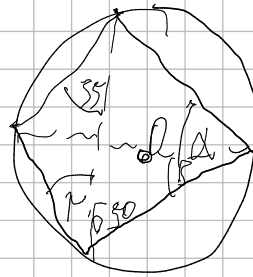


Teilung in Kreise

ad 7



c)



$$\begin{aligned} \gamma &= 75^\circ \\ \alpha &= 25^\circ \\ \beta &= 35^\circ \end{aligned}$$

Zad 7/237

$$360 : 16 = 22,5^\circ$$

a) $45 : 22,5 = 2$ $AC \vee AO$

b) $\beta = 112,5^\circ : 22,5 = 5$ $AF \vee AL$

d) $AC \vee AO$

Zad 2 str 237

a) $n=6$

$$L = \frac{2}{360} \cdot 2\pi r$$

$$\beta = 30^\circ \quad \alpha = 60^\circ$$

d) $\frac{\pi}{4} = \frac{\alpha}{360} \cdot 2\pi r$

$$\frac{\pi}{4} = \frac{\alpha}{360} \cdot 12\pi / : 12\pi$$

$$\frac{1}{4} = \frac{\alpha}{360} \quad \alpha = \frac{360}{4} = 90^\circ$$

$$30^\circ = \frac{\alpha}{4} \cdot \pi / : \pi$$

$$\alpha = 120^\circ$$

$$\beta = 31,75^\circ$$

Zad 3 str 238

a) $\alpha = 63^\circ$

b) $\beta = 102^\circ \quad \alpha = 57^\circ$

c) $\alpha = 37^\circ$
 $\beta = 64^\circ$

2. aufg 5

a) $\beta \quad \alpha = 2\beta$
 $\beta + 2\beta = 1170$
 $3\beta = 1170$
 $\beta = 390$
 $\alpha = 780$

c) $\beta \quad \alpha = 2\beta$

$$\frac{1}{3}\beta + \frac{1}{3}\beta = \frac{1}{3}\beta + \frac{2}{3}\beta = \frac{1}{3}\beta$$

$$\frac{4}{3}\beta = 48 / : \frac{4}{3}$$

$$\beta = 36 \cdot \frac{3}{4}$$

$$\beta = 27$$

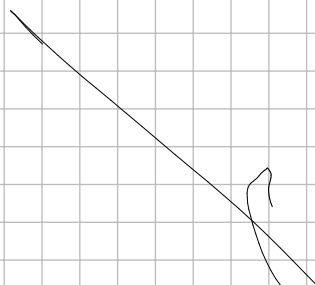
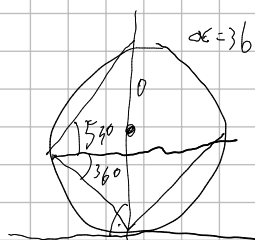
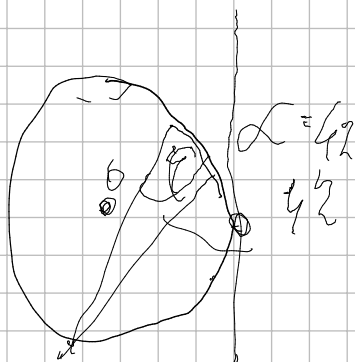
$$\alpha = 54$$

$$150$$

$$80 \quad \beta = 35$$

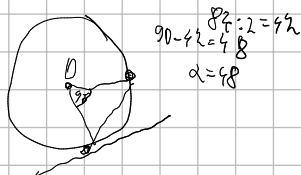
2. aufg 6

b)

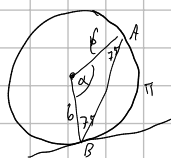


2. aufg 7

a)



b)



$$\pi = \frac{a}{360} \cdot 2 \cdot \pi \cdot r$$

$$\alpha = 30$$

$$90 - 30 = 60$$

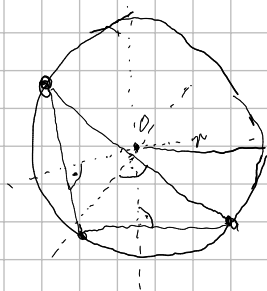
2. aufg 9

a) $4x = 12 \cdot 3$
 $x = 9$

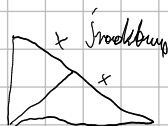
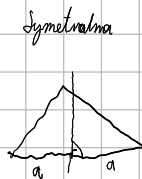
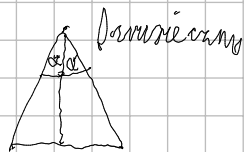
b) $4 \cdot 2 = 2x$
 $x = 4$

c) $5 \cdot 7 = 11$
 $(11-x) \cdot 2 = 6 \cdot 4 \quad \Delta =$
 $11x - x^2 = 24$
 $x^2 - 11x + 24 = 0$

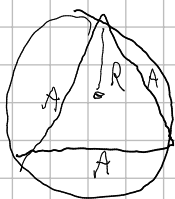
Trójkąt opisany na trójkącie



Przecięcie symetrycznych boków trójkąta wyznacza środek ciężkości, opisanego na trójkącie



Trójkąt opisany na tr. równobocznym



$$h = \frac{a\sqrt{3}}{2} \quad R = \frac{a\sqrt{3}}{3} \quad P = \frac{a^2\sqrt{3}}{4}$$

$$R = \frac{2}{3} h = \frac{2}{3} \cdot \frac{a\sqrt{3}}{2} = \frac{a\sqrt{3}}{3}$$

Pole trójkąta



$$P = \frac{a \cdot k \cdot c}{2R}$$

Ćw 3/240

a)



$$6 + \sqrt{3} = \frac{a^2\sqrt{3}}{4}$$

$$a = 16$$

$$R = \frac{a\sqrt{3}}{3} = \frac{16\sqrt{3}}{3}$$

$$P = \pi R^2 = \pi \cdot \left(\frac{16\sqrt{3}}{3}\right)^2 = \frac{4128\pi}{3}$$

Ćw 3b



$$6 = \frac{a\sqrt{3}}{3}$$

$$18 = a\sqrt{3}$$

$$\frac{18}{\sqrt{3}} = a$$

$$h = \frac{6\sqrt{3} \cdot \sqrt{3}}{2} = 9$$

$$P = \frac{6\sqrt{3} \cdot 9}{2} = \frac{54\sqrt{3}}{2} = 27\sqrt{3}$$

$$\frac{18\sqrt{3}}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = 6\sqrt{3}$$

Ćw 4b



$$r = 4,5$$

$$18 = \frac{4 \cdot 18}{2}$$

$$32 = 4 \cdot h$$

$$h = 8$$

$$L = 2 \cdot 45^\circ$$

$$L = 9^\circ$$

Ćw 5b



$$10^2 = (3x)^2 + x^2$$

$$100 = 9x^2 + x^2$$

$$100 = 10x^2$$

$$10 = x^2$$

$$x = \sqrt{10}$$

$$Okr = \sqrt{10} + 10$$

Ćw 7b



$$r^2 = 9 + 3^2$$

$$r = 3\sqrt{2}$$

$$3^2 + x^2 = 5^2$$

$$9 + x^2 = 25$$

$$x = 4$$



$$3^2 + h^2 = 5^2$$

$$h = 4$$

$$x = 4$$

$$r = \sqrt{10}$$

Ćw 9



$$P_1 = \frac{1}{2} \cdot 5 \cdot 3 = 7,5$$

$$R = \frac{5 \cdot 3}{4} = 3,75$$

$$\frac{P_1}{R} = \frac{7,5}{3,75} = 2$$

Ćw 10/242

b)

$$84 = \frac{13 \cdot 17 \cdot 15}{1R}$$

$$84 = \frac{2720}{2R}$$

$$R = 8 \frac{1}{2}$$

Zad 1 str 2 43

a)



$$P = \frac{a \cdot h}{2}$$

$$P = a \cdot p$$

$$r = 2$$

c)



$$Okr = 6$$

$$2x + x\sqrt{2} = 6$$

$$x(2 + \sqrt{2}) = 6 / (2 + \sqrt{2})$$

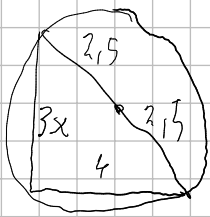
$$x = 6 - 3\sqrt{2}$$

$$x\sqrt{2} = 6\sqrt{2} - 6$$

$$r = 3\sqrt{2} - 2$$

Zad 2

$$P_0 = 6,25\pi$$



$$6,25\pi = \pi r^2 / \pi$$

$$6,25 = r^2 / \pi$$

$$r = 2,5$$

$$P_0 = \frac{3x \cdot 4}{2}$$

$$(3x)^2 + (4x)^2 = 5^2$$

$$25x^2 = 25$$

$$x = 1$$

Zad 5



$$b^2 = a^2 + c^2$$

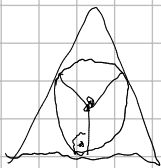
$$b^2 = 16 + 9$$

$$b^2 = 25$$

$$b = 5$$

t: okrąg wpisany w trójkąt

Okrąg wpisany w trójkąt

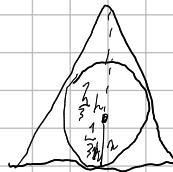


W każdym trójkącie można wpisać trójkąt



Środek trójkąta leży na przecięciu dwusiecznych

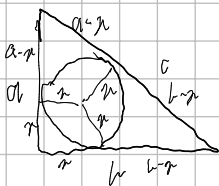
Okrąg wpisany w trójkąt równoboczny



$$r = \frac{1}{3} h_0$$

$$r = \frac{1}{3} \cdot \frac{a\sqrt{3}}{2} = \frac{a\sqrt{3}}{6}$$

Okrąg wpisany w tr. prostokątny



W. przeciętych trójkąt

$$c = (a-r) + (b-r)$$

$$c = a + b - 2r$$

$$r = \frac{a+b-c}{2}$$



$$P_0 = \frac{a+b+c}{2} \cdot r$$

Ćw 2

a) $\frac{18\sqrt{3}}{6} = 3\sqrt{3}$

$$2\pi r = 2\pi \cdot 3\sqrt{3} = 6\sqrt{3}\pi$$

b) $2\pi r = 10\pi$
 $r = 5$

$$5 = \frac{a\sqrt{3}}{6}$$

$$30 = a\sqrt{3} \cdot \sqrt{3}$$

$$a = \frac{30}{\sqrt{3} \cdot \frac{\sqrt{3}}{2}} = \frac{30\sqrt{3}}{3} = 10\sqrt{3}$$



Ćw 3



a) $L = 2\pi r = 2\pi \cdot 2\sqrt{3} = 4\sqrt{3}\pi$

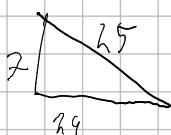
$$r = \frac{18\sqrt{3}}{6} = 3\sqrt{3}$$

$$R = 3\sqrt{3} \cdot 2 = 6\sqrt{3}$$

$$l = 2\pi r = 8\sqrt{3}\pi$$

$$L + l = 4\sqrt{3}\pi + 8\sqrt{3}\pi = 12\sqrt{3}\pi$$

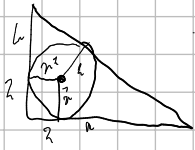
Ćw 4 u)



$$c = 25$$

$$r = \frac{a+b-c}{2}$$

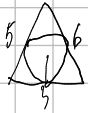
$$r = \frac{7+24-25}{2} = 2$$



$$\begin{aligned} 10 &= b - 2 + a - 2 & 14 &= b + a \\ 10^2 &= b^2 + a^2 & 14 - b &= a \\ 10^2 &= b^2 + (14 - b)^2 & 14 - 6 &= 8 \\ 10^2 &= b^2 + 196 - 28b + b^2 & 14 - 8 &= 6 \\ 10^2 &= 2b^2 - 28b + 196 & & \\ 2b^2 - 28b + 96 &= 0 & & \end{aligned}$$

$$\begin{aligned} \Delta &= 16 & \Delta_7 &= \frac{28 \cdot 4}{4} = 8 \\ \sqrt{\Delta} &= 4 & b_2 &= \frac{28 \pm 8}{4} = 6 \end{aligned}$$

Ćw 7



$$\begin{aligned} 2\pi r &= 3\pi & P &= \pi \cdot r \\ r &= \frac{3}{2} & P &= \left(\frac{5+5+6}{2} \right) \cdot \frac{3}{2} \\ & & P &= 8.14 = 11 \end{aligned}$$

Ćw 10

$$\begin{aligned} x &= 12 \\ y &= 10 \end{aligned}$$



$$P = \frac{a+b+c}{2} \cdot r$$

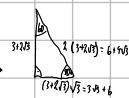
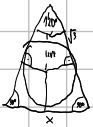
$$h = 8$$

$$\begin{aligned} a^2 + 6^2 &= 10^2 \\ a &= 8 \end{aligned}$$

$$P = \frac{1}{2} \cdot 10 \cdot 8 = 40$$

Ćw 11 str 246

a)



$$x = (12\sqrt{2} - 12) = 12\sqrt{2} - 12$$

Zad 2

a)



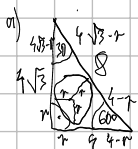
$$\begin{aligned} R &= \frac{a\sqrt{3}}{6} & P &= \frac{(a\sqrt{3})^2 \sqrt{3}}{4} = \frac{98\sqrt{3}}{4} = 12\sqrt{3} \\ r &= \frac{a\sqrt{3}}{6} / 3 & & \\ 12 &= a\sqrt{3} / \sqrt{3} & & \\ \frac{12 \cdot \sqrt{3}}{\sqrt{3}} &= 4\sqrt{3} & & \end{aligned}$$

b)

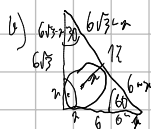


$$\begin{aligned} R &= \frac{a\sqrt{3}}{6} & 2r &= 12\sqrt{3} - 12 \\ R &= \frac{a\sqrt{3}}{6} & 2r &= 12\sqrt{3} - 12 \end{aligned}$$

Zad 3 str 247



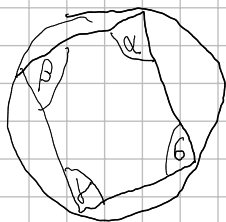
$$\begin{aligned} 4\sqrt{3} - r + 4r &= 8 \\ 2r &= 8 - 4\sqrt{3} \\ r &= 2\sqrt{3} - 2 \end{aligned}$$



$$\begin{aligned} 6\sqrt{3} - r + 6r &= 12 \\ 2r &= 12 - 6\sqrt{3} \\ r &= 3\sqrt{3} - 3 \end{aligned}$$

Zad 5

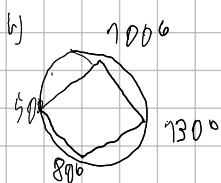
t: okrąg opisany na czworokacie



Środek okręgu
= przecięcie symetrycznych boków czworokąta

$$\alpha + \gamma = \beta + \delta = 180^\circ$$

Ćw 3/249



Ćw 46/249



Ćw 6



$$\begin{aligned} \alpha &= 36^\circ \\ \gamma &= 144^\circ \\ \beta &= 72^\circ \\ \delta &= 36^\circ \end{aligned}$$

Zad 1



$$\alpha + 3\alpha = 2\alpha + 4\alpha$$



$$3\alpha + 3\alpha = 6\alpha + 2\alpha$$

git

Zad 2



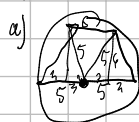
$$\begin{aligned} (10^\circ)^2 + (10^\circ)^2 &= 100 \\ 2 \cdot 10^\circ &= 200 \\ 10^\circ &= 100 \\ 32 \cdot 21 &= 768 (3) \end{aligned}$$

Zad 3



$$\begin{aligned} a &= \frac{3}{\sqrt{3}} \cdot \frac{\sqrt{3}}{2} = \sqrt{3} \\ p &= 2\sqrt{3} \\ p &= (2\sqrt{3})^2 = 12 \end{aligned}$$

Zad 4



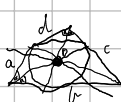
$$\begin{aligned} 5^2 + 5^2 &= x^2 \quad x^2 + 5^2 = y^2 \\ x &= 5 \quad y &= 2\sqrt{5} \\ \text{Okrąg} &= 16 + 4\sqrt{5} \end{aligned}$$

Zad 5



$$\begin{aligned} 3^2 + 4^2 &= 5^2 \\ a &= 4 \\ b^2 + 4^2 &= 5^2 \\ b &= 3 \\ p &= \frac{3^2 + 4^2}{2} = 12.5 \end{aligned}$$

T: Okrąg wpisany w czworokąt



Przecięcie dwusiecznych czworokąta = środek okręgu
 $a + c = b + d$

Üw 5/25f

b)



$$r + R = h + d$$

$$12 = h + d$$

$$h = 4 = 2r$$

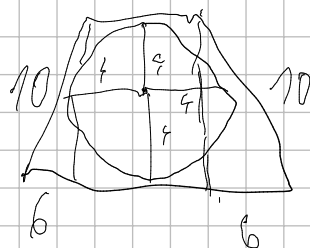
$$P = \frac{(b+d) \cdot h}{2}$$

$$P = \frac{12 \cdot 4}{2}$$

$$P = 24$$

$$L = 24$$

c)

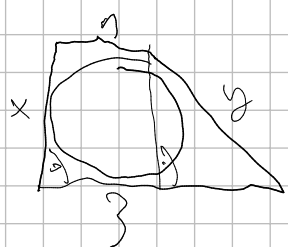


$$P = \frac{(x+x+12) \cdot 8}{2} = 8x + 48 = 912$$

$$x + x + 12 = 20$$

$$x = 4$$

2. Aufl.

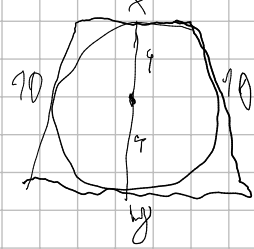


$$y^2 = x^2 + 4^2$$

$$x + y = 12$$

$$x = 4 + y$$

2. Aufl. 2. Aufl. 2. Aufl.



$$10 + 10 = 20$$

$$P = \frac{20 \cdot 8}{2} = 80$$

2. Aufl.

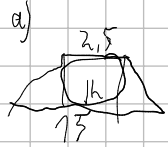


h=2

$$h = 2\sqrt{2} - 2$$

$$|AB| = 2\sqrt{2} + 2$$

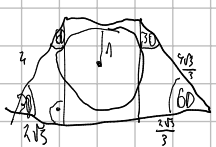
2. Aufl.



$$P = \frac{(1.5 + 2.5) \cdot 6}{2} = 52.5$$

$$L = 2.15 + 1.5 + 17.5 = 35$$

2. Aufl.



$$\frac{6\sqrt{3}}{3} + 4 = 2h + 2\sqrt{3} + \frac{6\sqrt{3}}{3}$$

$$4\sqrt{3} + 12 = 6h + 6\sqrt{3} + 4\sqrt{3}$$

$$6h = 12 - 4\sqrt{3}$$

$$h = 2 - \frac{2\sqrt{3}}{3}$$

$$|AB| = 2 \cdot \frac{4\sqrt{3}}{3} + 2\sqrt{3} + \frac{6\sqrt{3}}{3}$$

$$|AB| = 2 \cdot \frac{4\sqrt{3}}{3} + 2\sqrt{3} + \frac{6\sqrt{3}}{3}$$

2. Aufl.

a)



$$\sin 60^\circ = \frac{h}{2}$$

$$2 \sin 60^\circ = h$$

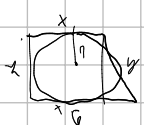
$$2 \cdot \frac{\sqrt{3}}{2} = h$$

$$h = \sqrt{3}$$

$$x = \frac{h}{2} = \frac{\sqrt{3}}{2}$$

$$P = \frac{(x+x) \cdot h}{2} = \frac{3}{4}$$

2. Aufl.



$$6+x = 2+y$$

$$x = y - 4$$

$$(6-x)^2 + 4^2 = y^2$$

$$36 - x^2 + 4 = y^2$$

$$36 - (y-4)^2 + 4 = y^2$$

$$36 - y^2 + 8y + 16 + 4 = y^2$$

$$56 - y^2 + 8y = y^2$$

$$56 - 8y = 2y^2$$

$$28 - 4y = y^2$$

$$y = \sqrt{28 - 4y}$$

$$y =$$

b)



$$x = 2$$

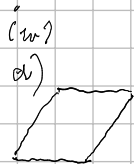
$$h = 4$$

$$h = 4 \sin 30^\circ$$

$$4 = 4 \cdot \frac{1}{2} \quad \alpha = 30^\circ$$

$$P = 32$$

T: Wielokąty foremne



Stwierdzenia



$$R = a$$

$$r = r_{\Delta} = \frac{a\sqrt{3}}{2}$$

$$P = 6 \cdot P_{\Delta} = 6 \cdot \frac{a^2 \sqrt{3}}{2} = \frac{3a^2 \sqrt{3}}{2}$$

(wz) a) $150\sqrt{3} = \frac{3a^2 \sqrt{3}}{2}$
 $300\sqrt{3} = 3a^2 \sqrt{3} / : \sqrt{3}$
 $100\sqrt{3} = 3a^2 \sqrt{3} / : \sqrt{3}$
 $100 = a^2$
 $a = 10$

$R = 10$
 $d = 2R = 20$

b) $P = 32\pi$
 $32\pi = \pi r^2$
 $r = 4\sqrt{2}$
 $P_{\Delta} = \frac{3(4\sqrt{2})^2 \sqrt{3}}{2} = 48\sqrt{3}$

(wz) a) $a = \frac{F_2}{6} \approx 9$ $r = \frac{a\sqrt{3}}{2}$

$$P = \pi \left(\frac{a\sqrt{3}}{2} \right)^2 = \frac{\pi a^2 \sqrt{3}}{4}$$

b) $l = 12.7$
 $r = r_{\Delta} = 6$



$6 = \frac{a\sqrt{3}}{2}$
 $12 = a\sqrt{3} / : \sqrt{3}$
 $\frac{12}{\sqrt{3}} = a$

$d = 2a = 8\sqrt{3}$ $d_2 = 2r = 12$

zad 1/258



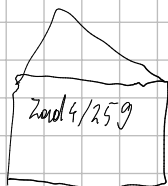
$360 : 10 = 36$

zad 2/258

$n = \frac{S}{P_{\Delta} \cdot t} = 6$

a) $\frac{240}{240} + 2 = 4 + 2 = 6$

b) $\frac{200}{200} + 2 = 7$



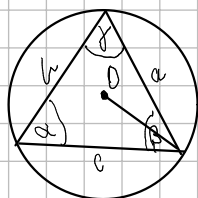
zad 3



$P_{\Delta} = \pi a^2$

T: Twierdzenie sinusów

Twierdzenie sinusów działa na dowolnych trójkątach i dotyczy zależności między bokami i kątami



$$\frac{a}{\sin \alpha} = \frac{b}{\sin \beta} = \frac{c}{\sin \gamma} = 2R$$

Przykład boku długości a i b



$$\frac{a}{\sin 120^\circ} = \frac{b}{\sin 150^\circ} = \frac{3}{\sin 45^\circ}$$

$\frac{a}{\frac{\sqrt{3}}{2}} = \frac{3}{\frac{1}{2}}$

$\frac{\sqrt{3}}{2} a = 3 \sqrt{3} / : \sqrt{3}$

$a = \frac{3\sqrt{3}}{\sqrt{3}} = 3$

$\frac{b}{\sin 110^\circ} = \frac{3}{\sin 45^\circ}$

$\frac{b}{0.76604} = \frac{3}{0.7071}$

$b = \frac{0.76604}{0.7071} = 1.1$

1/262

b) $\alpha = 30^\circ$ $\beta = 105^\circ$ $\gamma = 45^\circ$



$\frac{a}{\sin 100^\circ} = \frac{b}{\sin 105^\circ} = \frac{P}{\sin 150^\circ}$

$\frac{a}{2} = \frac{P}{\sqrt{2}}$

$a \sqrt{2} = P$

$a = \frac{P}{\sqrt{2}} = 4\sqrt{2}$

$\frac{b}{\sin 120^\circ} = \frac{P}{\sqrt{2}}$

$b = \frac{P}{\sin 120^\circ} = 8$

$b = 10.8$

in 2

1) $a=7$ $b=6$ $\alpha=80^\circ$



$$\frac{7}{\sin 80^\circ} = \frac{6}{\sin \beta}$$

$$\frac{7}{0.986} = \frac{6}{\sin \beta}$$

$$7 \sin \beta = 6 \cdot 0.986$$

$$7 \sin \beta = 5.916$$

$$\sin \beta = 0.846$$